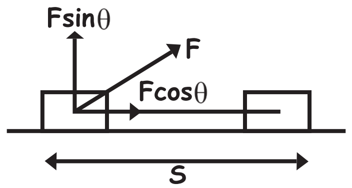
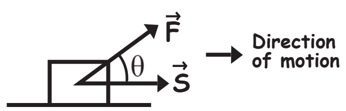
WORK ENERGY & POWER

WORK
Work is said to be done when a force applied on the body displaces the body through a certain distance

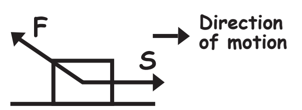
WORK DONE BY CONSTANT FORCE


$W = F\cos\theta \times S$
$= FS\cos\theta = \vec{F}.\vec{S}$

NATURE OF WORK DONE
1) Positive work ($0^\circ \le \theta < 90^\circ$)



2) Negative work ($90^\circ < \theta \le 180^\circ$)

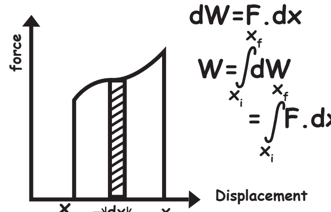


2) Zero work
Work done becomes 0 for three conditions
1. Force is perpendicular to displacement
2. if there is no displacement
3. if there is no force acting on the body

WORK DONE BY VARIABLE FORCE
$d W = \vec{F}.\vec{ds}$
$W = \int \vec{F}.\vec{ds} = \int F ds \cos\theta$

in terms of rectagular components
$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
$\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$
$W = \int F_x dx + \int F_y dy + \int F_z dz$

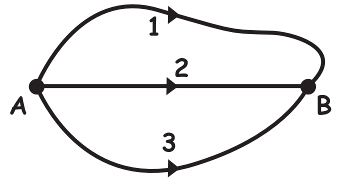
Graphical representation of work done



$dW = F.dx$
$W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F.dx$

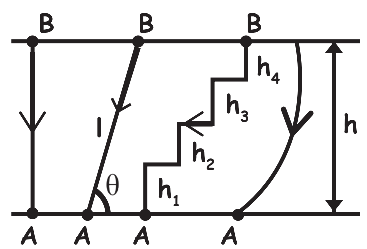
W= Area under F-x graph

WORK DONE BY CONSERVATIVE & NON CONSERVATIVE FORCE
Conservative: work done doesnot depend on path followed
Non-conservative: work depends on the path followed



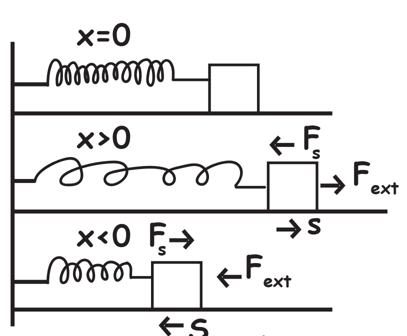
- $W_{A\to B}$(Path 1) $= W_{A\to B}$(Path 2) $= W_{A\to B}$(Path 3)
  (for conservative force)
- $W_{A\to B}$(Path 1) $\ne W_{A\to B}$(Path 2) $\ne W_{A\to B}$(Path 3)
  (for non conservative force)

Note :
Work done for a complete cycle for a conservative force is zero

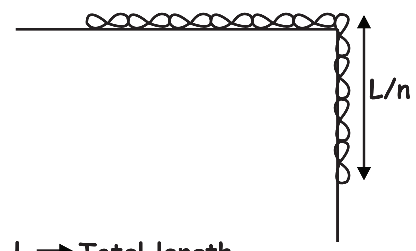
WORK DONE BY DIFFERENT FORCES


$W_1 = mgh = mgh$
$W_2 = mg \times l \sin\theta = mg \times l \times \frac{h}{l}$
$= mgh$
$W_3 = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$

<u>Work done by spring force</u>
magnitude of spring force, $F_s = -kx$



$W_s = \int \vec{F_s}.\vec{dx} = -\int_{x_i}^{x_f} kx dx$
$= -\frac{1}{2}K(x_f^2 - x_i^2)$

WORK DONE IN PULLING THE CHAIN


L → Total length
$(1/n)^{th}$ part of length hanging
M → Mass of chain
Work done in pulling the hanging portion back on the table
$W = \frac{MgL}{2n^2}$

ENERGY
- Capacity of doing work
- Scalar quantity
- Dimension ML^2T^{-2}

Relation between different units
$1 eV = 1.6 \times 10^{-19}$ Joules
$1 kWh = 3.6 \times 10^6$ Joules
$1 calorie = 4.18$ Joules
1 Joule $= 10^7$ erg

Kinetic Energy
- Energy possessed by virtue of motion

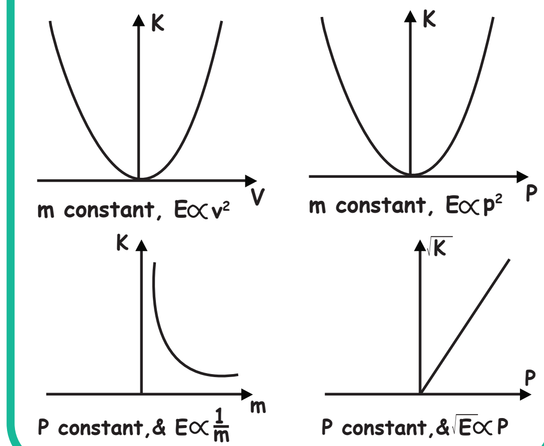
$K.E = \frac{1}{2}mv^2$

- Always positive
- Depends on frame of reference

<u>Work Energy Theorm</u>
Change in kinetic energy of a body is equal to network done on the body

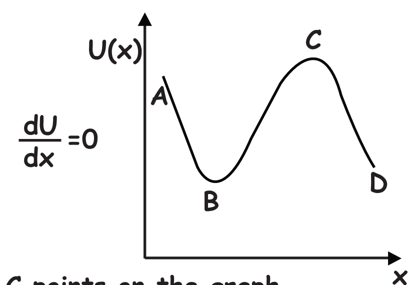
$K_2 - K_1 = \int \vec{F}.\vec{dr}$

RELATION OF KINETIC ENERGY WITH OTHER QUANTITIES
Linear momentum:- $P = \sqrt{2mK}$
Variation of graph of kinetic Energy



POTENTIAL ENERGY
- Defined only for conservative force
- Energy possessd by a body by virtue of its position/configuration
- Can either be positive, negative or zero according to point of reference
- Force always acts from higher potential to lower potential

Identifying forces with the help of potential energy
1) Force opposing the motion:-
On increasing x, if U increases
$\frac{dU}{dx}$ = positive
(BC portion of graph)
2) Force supporting the motion:-
On increasing x, if U decreases
$\frac{dU}{dx}$ = negative
(AB portion of graph)
3) Zero force:-
On increasing x, if U doesnot change

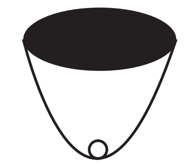


B,C points on the graph

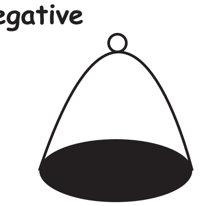
Types of Potential Energy
-Elastic Potential Energy
-Electric Potential Energy
-Gravitational Potential Energy

·Types of equilibrium
If net force acting on a particle is zero it is said to be in equillibrium

STABLE
- If particle displaced from equillibrium position, force acting will try to bring the particle back to the equilibrium position
- Potential energy is minimum at stable equilibrium
- $F = \frac{-dU}{dx} = 0$
- $\frac{d^2U}{dx^2}$ = positive

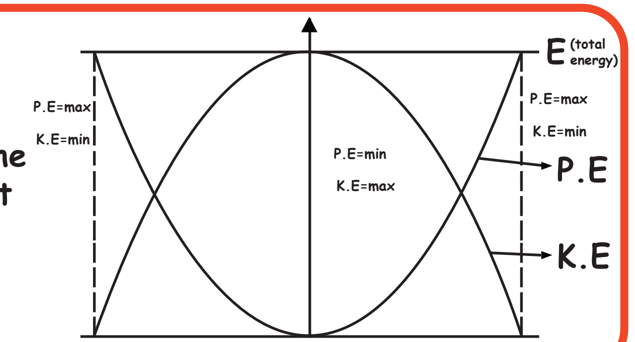


UNSTABLE
- If particle displaced from equillibrium position, force acting on it tries to displace it further away from equillibrium position
- Potential energy is maximum at unstable equilibrium
- $F = \frac{-dU}{dx} = 0$
- $\frac{d^2U}{dx^2}$ = negative



NUETRAL
- If particle is slightly displaced from equillibrium, then it does not experience a force or continues to be in equillibrium
- Potential energy is constant
- $F = \frac{-dU}{dx} = 0$
- $\frac{d^2U}{dx^2} = 0$

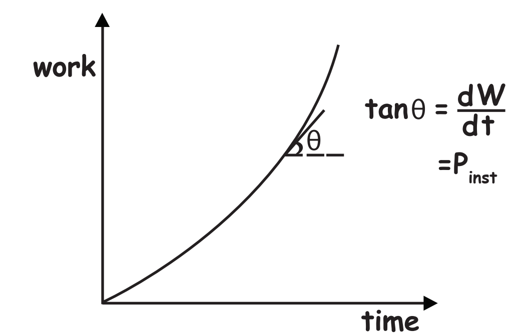
CONSERVATION OF ENERGY
For an isolated system or body in the presence of only conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion
K.E+P.E=constant



POWER
- Rate at which body does work
- Average power$(P_{av}) = \frac{W}{t}$
- Instanteneous power

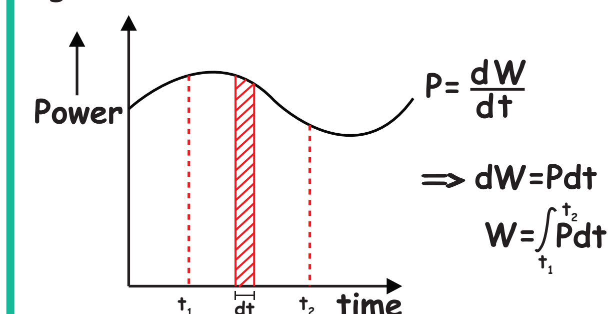
$(P_{inst}) = \frac{dW}{dt} = \frac{\vec{F}.\vec{ds}}{dt} = \vec{F}.\vec{v}$

Relation between units:
- 1 watt=1joule/sec $= 10^7$ erg/sec
- 1 HP=746watt, 1MW $= 10^6$ watt
- 1 KW$= 10^3$ watt
- If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$
- Unit of power multiplied by time always gives work
  1 KWh$= 3.6 \times 10^6$ Joules
- Slope of work-time curve gives instanteneous power



$\tan\theta = \frac{dW}{dt}$
$= P_{inst}$

- Area under power-time graph gives work done



$P = \frac{dW}{dt}$
$\Rightarrow dW = Pdt$
$W = \int_{t_1}^{t_2} Pdt$

W=Area under P-t graph

Position and velocity in terms of power (P=constant)

1) Velocity, $V=\left[\frac{2Pt}{m}\right]^{1/2}$

2) Position, $S=\left[\frac{8P}{9m}\right]^{1/2} t^{3/2}$

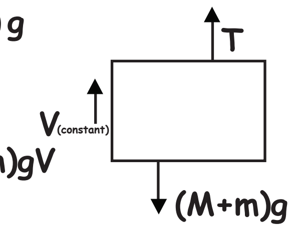
Power delivered by an elevator

$a=0,\ T=(M+m)g$

$\vec{P}=\vec{T}.\vec{V}$

$=TV$

Power, $P=(M+m)gV$



Power of a water drawing pump

- Power, $P=\frac{dW}{dt}=\frac{dm}{dt}\left[gh+\frac{V^2}{2}\right]$
- h=height of water level

$\frac{dm}{dt} \Rightarrow$ mass flow rate of pump

$V \rightarrow$ velocity of the water outlet

- Power required to just lift water, V=0

$P=gh\left(\frac{dm}{dt}\right)$

Efficiency of pump

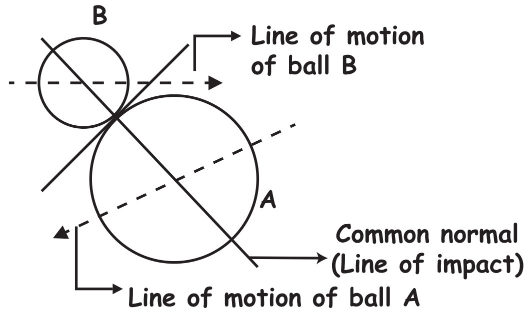
$$\mu = \frac{\text{Output Power}}{\text{Input Power}}$$

WORK ENERGY & POWER

Collision is the event in which impulsive force acts between two or more bodies which results in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact



Coefficient of restitution (e)

$$e = \frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$$

$$= \frac{\text{Relative velocity along the line of impact after collision}}{\text{Relative velocity along the line of impact before collision}}$$

Conditions

1. For elastic collision: e=1
2. For inelastic collision: e<1
3. For perfectly inelastic collision: e=0

Classification

- Based on conservation of kinetic energy
  - Perfectly elastic
  - Inelastic
  - Perfectly inelastic
- Based on direction of colliding bodies
  - Oblique
  - Head-on/ one dimensional

Perfectly elastic collision

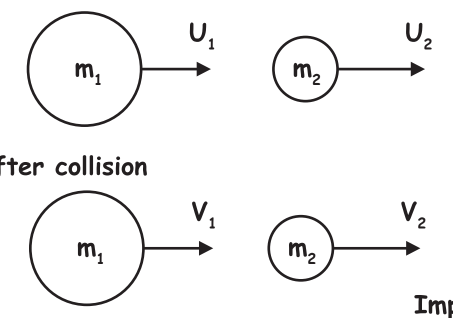
K.E before and after collision is same

Inelastic collision

K.E before and after collision is not same

Head-on collision / One dimensional collision

Initial velocities of the bodies are along the line of impact.

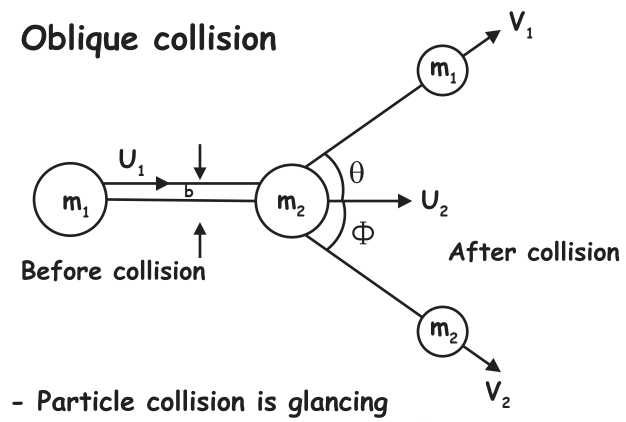


Impact parameter b=0

COLLISION

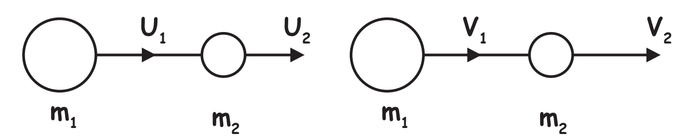
If initial velocities of the bodies are not along the line of impact.

Oblique collision



- Particle collision is glancing
- Directions of motion after collision are not along initial line of motion
- Impact parameter $0<b<(r_1+r_2)$ where r_1, r_2 are radii of colliding bodies

Perfectly elastic Head-on collission



Velocity after collision

$$V_1 = U_1\left[\frac{m_1-m_2}{m_1+m_2}\right] + \frac{2m_2U_2}{m_1+m_2}$$

$$V_2 = U_2\left[\frac{m_2-m_1}{m_1+m_2}\right] + \frac{2m_1U_1}{m_1+m_2}$$

Special cases:

1) Projectile and target having same mass $m_1=m_2$, then $v_1=u_2, v_2=u_1$, the velocities get interchanged.

2) If massive projectile collides with a light target i.e. $m_1>>>m_2$, then $v_1=u_1, v_2=-u_2+2u_1$

3) If a light projectile collides with a very heavy target, $m_1<<m_2$, then $v_1=-u_1+2u_2, v_2=u_2$

Energy transfer from projectile to target

1) Fractional decrease in kinetic energy of projectile (If target is at rest)

$$\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1-m_2)^2+4m_1m_2}$$

Greater the difference in masses, less will be transfer of K.E and vice versa

Transfer of K.E will be maximum when difference in masses is maximum

If $m_2=nm_1$ $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

Inelastic collision

$$e = \frac{V_2-V_1}{U_1-U_2} \quad \frac{\text{Relative velocity of seperation}}{\text{Relative velocity of approach}}$$

Velocity after collision $V_1 = \frac{(1+e)\,m_2U_2}{m_1+m_2} + \frac{(m_1-em_2)U_1}{m_1+m_2}$

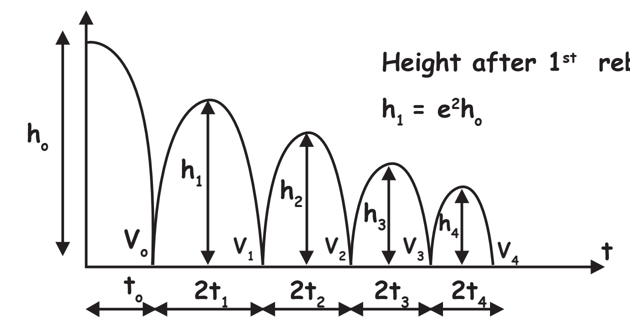
$V_2 = \frac{(1+e)\,m_1U_1}{m_1+m_2} + \frac{(m_2-em_1)U_2}{m_1+m_2}$

Ratio of velocities

$\frac{V_1}{V_2} = \frac{1-e}{1+e}$

Loss in kinetic energy $\Delta K = \frac{1}{2}\left[\frac{m_1m_2}{m_1+m_2}\right](1-e^2)(U_1-U_2)^2$

Rebounding of ball



Height after 1st rebound

$h_1 = e^2h_o$

Total height covered by the ball before it stops bouncing

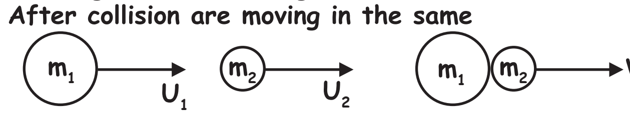
$H = h_o\left[\frac{1+e^2}{1-e^2}\right]$

Total time taken by the ball until ot stops bouncing $T = \left(\frac{1+e}{1-e}\right)\frac{2h_o}{g}$

Perfectly inelastic collision

Colliding bodies stick together

After collision are moving in the same



$V = \frac{m_1U_1+m_2U_2}{m_1+m_2}$

Loss in kinetic energy $\Delta k = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(U_1-U_2)^2$

Colliding bodies are moving in the opposite direction

$V = \frac{m_1U_1-m_2U_2}{m_1+m_2}$, Change in kinetic enargy $\Delta K = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(U_1-U_2)^2$